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Ballistic magnetoresistance and the Hall effect in a restricted geometry

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Abstract. We have measured the magnetoresistance and Hall resistance of an open-cross structure formed in a two-dimensional electron gas by electrostatic depletion. We observe effects similar to those seen by other authors in more closed geometries. We interpret our results using the classical model of Beenakker and van Houten. By measuring the ballistic transmission coefficient directly we are able to provide quantitative confirmation of the model.

There has been a great deal of interest recently in the electrical properties of structures which are small relative to the electron mean free path. The conductance of a single quantum point contact (QPC) is known to be quantised approximately in units of $2e^2/h$ [1, 2]. Also in low magnetic fields anomalous magnetoresistances, bend resistances and the quenching of the Hall effect have all been observed [3–5] in cross-shaped conductors. These phenomena have been interpreted in terms of 'classical' particle ballistics [6] as well as being subjected to a more rigorous quantum analysis [7]. We present in this paper a series of experiments on an open structure which has the topology of a cross. The structure is defined by electrostatic depletion underneath four narrow metallic gates at right angles to each other as shown schematically in figure 1. Note that the electron channels which lead to the intersection of the cross have a more open shape than in a conventional bar-shaped geometry. The electrostatic depletion gives rise to a smooth-walled channel. Thus, the scattering-off irregularities in the channel wall is expected to be very small. Such scattering is known to cause similar anomalies [8] to those reported here. Also, the more open geometry of our experiment means that we do not have to worry about effects in narrow wires. We are able to measure independently the ballistic transmission coefficients relevant for our structure which we are then able to correlate successfully with the magnetoresistance and Hall resistance.

The sample is fabricated on a GaAs/(AlGa)As modulation-doped heterostructure, which has a two-dimensional electron gas (2DEG) with an electron concentration of $2 \times 10^{11}\ \mathrm{cm}^{-2}$ (after illumination with infra-red light) and an electron mean free path of $\sim 10\ \mu\mathrm{m}$ at low temperatures. The four independent metallic gates of Ti/Au are

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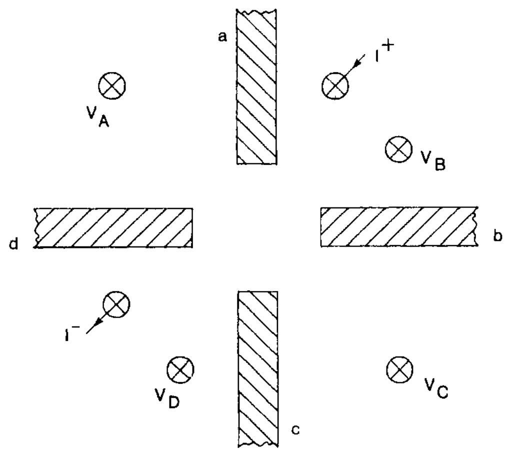

Figure 1. A schematic diagram of the open-cross structure. The hatched regions are metallic gates and are labelled a, b, c, d. Opposite gates are 0.8 μm apart. The ⊗ symbols represent electrical contacts in the 4 regions of 2DEG A, B, C, D.

deposited on the heterostructure using electron beam lithography and lift-off techniques. A plan view of the gate and contact geometry is shown in figure 1. By applying a sufficiently large negative bias with respect to the 2DEG, the regions below the gates can be fully depleted of electrons, thus defining the cross structure. As the bias is made more negative, the conducting channels between adjacent pairs of gates are constricted. This corresponds to a transition from an open-cross structure to four QPCs probing a central dot. All measurements were made with a four-wire AC resistance bridge with 10 nA excitations at a temperature of 100 mK.

The four gates are labelled a, b, c, d and the four 2DEG regions A, B, C, D as shown in figure 1. The current I is passed between region B and region D. For Hall measurements we define $R_H = (V_A - V_C)/I$ and for magnetoresistance $R = (V_B - V_D)/I$. The Hall effect and magnetoresistance for magnetic fields from -0.6 to 0.6 T are shown in figure 2 for a range of gate voltages, V_g. The same voltage is applied to all four gates. When the gates are earthed ($V_g = 0$) and hence do not effect the 2DEG, the magnetoresistance exhibits the normal Shubnikov–de Haas oscillations and the Hall resistance is linear in magnetic field until the quantised plateaux begin to form. We have no convincing explanation for the very small anomalies near $B = 0$ or for the slight asymmetry between negative and positive magnetic fields. However, we note that no Hall bar geometry is defined for zero gate voltage.

As the gate voltage is applied, several qualitative features emerge. As soon as the cross is defined ($V_g = -0.5$ V) the zero-field resistance is much larger, as one would expect, but there is a strong negative magnetoresistance and shoulders appear at $B \simeq \pm 0.15$ T. The corresponding Hall resistance flattens near $B = 0$ and also develops kinks at around ± 0.15 T. As V_g is made more negative, the shoulders in the magnetoresistance develop into definite peaks ($V_g \sim -1.3$ V) and eventually, before the cross pinches-off completely, the peaks become very pronounced ($V_g = -2.03$ V). The Hall resistance also develops structure. Near $B = 0$, the slope of the Hall resistance versus B curve quenches to zero ($V_g = -1.43$ V) and then reverses in sign ($V_g < -1.57$ V). The kinks become sharper and gradually a flat region develops above 0.25 T where the Hall resistance is independent of magnetic field ($V_g < -1.43$ V). In addition to these symmetric features there is also a certain amount of reproducible, though not necessarily symmetric, structure in both the magnetoresistance and Hall resistance. The structure becomes more apparent as the magnitude of V_g increases.

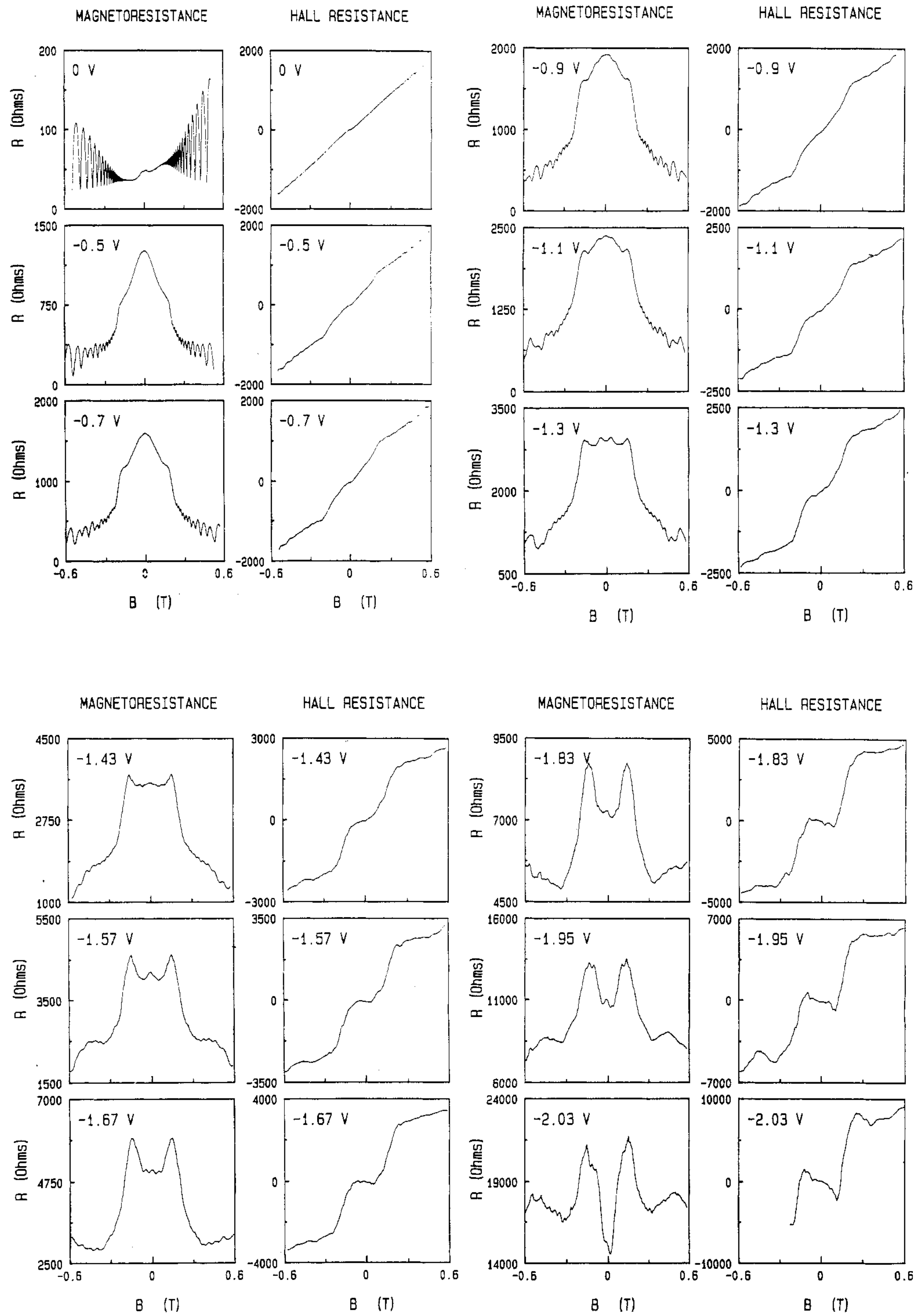

Figure 2. Magnetoresistance and Hall resistance at various gate voltages for $-0.6 < B < 0.6$ T at a temperature of 100 mK. In each case the voltage is applied to all four gates equally.

We now discuss and interpret these results. The quenching and reversal of the Hall effect have been interpreted in terms of classical electron ballistics [6] involving the effect of the classical Lorentz force and specular reflection off the gate potentials. The classical Hall voltage occurs because electrons are deflected by the magnetic field to one side of the sample. If no current is drawn then a potential difference must exist to counteract the effect of the magnetic field. The quenching of the Hall effect in a cross structure is thought to occur when an electron injected from one arm of the cross has its trajectory scrambled by repeated specular collisions with the confining electrostatic potential [6]. Despite the curvature of the trajectory introduced by the magnetic field, the scrambling makes it equally likely that the electron is deflected into either of the two Hall contacts so the Hall voltage disappears. Reversal of the Hall voltage can occur when electrons deflected, say, to the left by the magnetic field are reflected by elastic collisions into the right-hand contact and the Hall voltage will have the opposite sign to what one would expect [6].

At slightly higher magnetic fields the classical cyclotron radius at the Fermi energy, l_c, becomes sufficiently small to reduce the effect of reflections on the Hall voltage and essentially all the electrons injected into the cross are deflected by the magnetic field into the 'correct' Hall probe, i.e. the Hall voltage will have its usual sign. However, the number of electrons entering the Hall probe is determined by the number of conducting channels in the injection contact, N, or by the number of occupied Landau levels, N_L, whichever is the smaller. Thus we expect a region where, as B is increased, the Hall voltage is determined by N ($R_H = h/2e^2N$) and is independent of B. Eventually, as B is increased, N_L becomes less than N and the normal Hall resistance is regained.

According to Beenakker and van Houten [6], whether quenching or reversal occurs is quite sensitive to the geometric shape of the electrostatic potential profile of the gates as seen by the electrons. Our results appear to show a monotonic change in R_H as V_g is made more negative. The flat region of R_H, or last plateau, is predicted to occur when the classical cyclotron radius is *smaller* than the bend radius r_{min}, defined as the largest radius for an electron injected from one contact to pass ballistically into an adjacent one, but larger than half the width of a contact. For our devices this corresponds to $0.15\,T < B < 0.4\,T$ at $V_g = -0.5\,V$ and $0.15\,T < B < 1.3\,T$ at $V_g = -2\,V$ consistent with the experimental data. As an independent check of this, we have measured the ballistic transmission coefficient T between regions B and A, using the method described by Main *et al* [9]. This method utilises the quantised conductance of an individual QPC to determine the ballistic transmission through two QPCs in series and is described briefly below.

The resistance between 2DEG regions A and C, R_{AC}, is measured by keeping gate c at ground potential while sweeping the negative bias applied to gates a, b and d. This means that gate c does not act on the 2DEG and gates a, b, d form two perpendicular QPCs. The geometry is described in more detail in [9]. Similarly R_{BC} and R_{AB} are obtained. R_{AC} and R_{BC} are the resistances of the individual QPCs and show stepped behaviour as a function of gate voltage. R_{AB} is the resistance of the two perpendicular QPCs in series. The four-terminal resistances measured are converted to equivalent two-terminal measurements by addition of the constant h/e^2N_L, where N_L is the number of occupied Landau levels in the unconfined 2DEG. The normalised transmission coefficient, T, can be written in terms of the corrected two-terminal resistances using the Büttiker formalism [9, 11]

$$T = (2e^2/h)N(R_{AC} + R_{BC} - R_{AB})$$

where N is the number of conducting channels in each QPC.

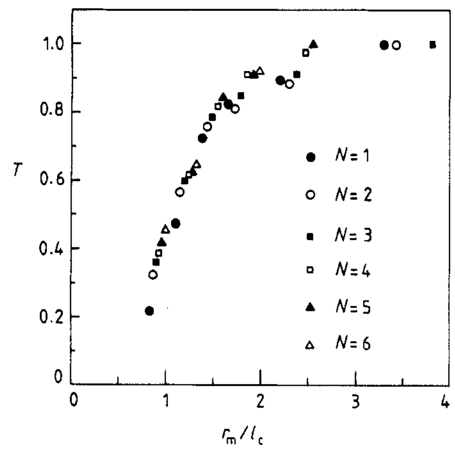

Figure 3. A plot of ballistic transmission coefficient, T, between regions B and A as a function of the ratio of the maximum radius, r_m, of a classical electron trajectory between regions B and A to the classical cyclotron radius at the Fermi energy l_c, for magnetic fields up to 0.6 T and for the number, N, of occupied 1D subbands in a QPC of 1 to 6.

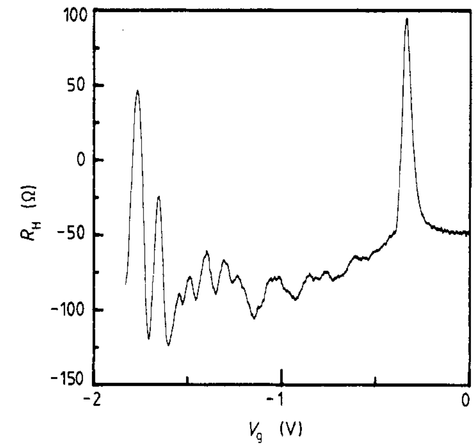

Figure 4. The Hall configuration resistance, R_H, plotted as a function of gate voltage for $B = 0$ T at $T = 100$ mK.

Figure 3 shows T plotted as a function of the ratio of the maximum radius, r_m, of a classical electron trajectory between regions B and A to the cyclotron radius at the Fermi energy, $l_c = mv_F/eB$, for magnetic fields up to 0.6 T and for N between 1 to 6. The results shown in figure 3 are a striking confirmation of the simple picture of electron ballistics [6]. There is a steep increase in T for $r_m/l_c \sim 1$. Values of T corresponding to N from 1 to 6 appear to lie on the same curve indicating that even for the narrowest QPCs, where the width is only half a Fermi wavelength, diffraction effects are not important. For our structure $r_m/l_c \sim 1$ for $B \simeq 0.15$ T which is in excellent agreement with the onset of the flat regions of Hall resistance as seen in figure 2.

The zero-field maximum in the magnetoresistance which occurs for $-1.1\text{ V} \leq V_g \leq -0.5$ V is simply due to the restriction on the 2DEG formed by the gates and the negative magnetoresistance associated with a four-wire measurement of a constricted channel [10]. The slope of the negative magnetoresistance increases for $B \geq 0.2$ T and at more negative V_g, positive bumps in the magnetoresistance occur at ± 0.15 T. These are due to the destruction of collimation due to the Lorentz force and the beginning of the formation of edge states [12]. At $B = 0$ some electrons can travel ballistically through the device due to the collimating influence of the constrictions. The application of a magnetic field destroys this collimation totally when the classical cyclotron radius is smaller than the bend radius, which occurs at the same field as the establishment of the flat region of Hall resistance, once again consistent with experimental results. The fine structure which occurs is probably due to ballistic resonances but without explicit detailed calculation of the potential profiles it is difficult to identify them.

The asymmetry in the fine structure reflects the slight asymmetry in our cross structure which arises from processing misalignment. We demonstrate this directly in figure 4 where we illustrate the Hall resistance, i.e. the resistance in a Hall configuration, $R_H = (V_A - V_C)/I$, for $B = 0$ as V_g is varied between 0 and −2 V. For a bulk conductor this is zero. The fluctuations we measure are due to small differences in the transmission probability into each side contact for electrons injected from a current contact. The peak at $V_g = -0.34$ is where the cross structure is first defined by the gates. The largest fluctuations occur when the cross is close to pinch-off. Note that since all our measurements are performed using the fundamental frequency in an AC bridge we are not sensitive to the thermopower fluctuations which have been measured in a similar geometry and for which $V_A - V_C$ would be an even function of the current [13]. Note that the changes in the Hall resistance due to the gate voltage in zero-magnetic field are small compared to the changes introduced by a magnetic field.

In conclusion, we have investigated the Hall resistance and magnetoresistance of an open-cross structure where scattering from inhomogeneities in the wire edges is not present. The main features in these measurements can be related to the rapid increase with magnetic field of the ballistic transmission through perpendicular QPCs in a magnetic field of >0.15 T. Our measurements provide quantitative confirmation of the classical picture of Beenakker and van Houten.

Acknowledgments

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